Polynomial Identities of the Weyl Algebra

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Abstract.

Let $\mathbb{F}$ be a field and X an enumerable set. The *free associative algebra* of X over $\mathbb{F}$ is the algebra $\mathbb{F}\langle X\rangle$ of polynomials on the non-commutative variables $x \in X$. Let A be an $\mathbb{F}$-algebra and $f = f(x_1, \ldots, x_n) \in \mathbb{F}\langle X\rangle$. We say that $f \equiv 0$ is a *polynomial identity of* A if $f(a_1, \ldots, a_n) = 0$ for all $a_1, \ldots, a_n \in A$. It is obvious that the trivial polynomial is an identity for any algebra. Than, if A satisfies a non-trivial identity $f \equiv 0$, we say that A is a *PI-algebra*.

We are interested in studying polynomial identities for the Weyl algebra A_n. Define the algebra A_1 as the non-commutative algebra over $\mathbb{F}$ generated by x and y with the relation $yx = xy + 1$, i. e., $A_1 = \dfrac{\mathbb{F}\langle x, y\rangle}{(yx - xy - 1)}$. For $n > 1$, we define the n-th Weyl algebra recursively as $A_n = A(A_{n-1})$. It is a known fact that if the field $\mathbb{F}$ has characteristic zero, then the Weyl algebra admits no nontrivial identities (see [3]). However, in case $\mathbb{F}$ has characteristic $p > 0$, there do exist identities for the Weyl algebra. For example, the standard polynomial St_4 (respec. St_6) is an indentity of A_1 in case $p = 2$ (resp. $p = 3$), being

$$St_m(x_1, \ldots, x_m) = \sum_{\sigma \in Sm} sgn(\sigma) x_{\sigma(1)} \ldots x_{\sigma(m)}$$

the standard polynomial of degree m. In this work, we study polynomial identities in the case where the characteristic of the field is positive.

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